

Learning to Teach Mathematics for Understanding: The Role of Reflection

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This paper explores how a group of 11 preservice secondary mathematics teachers developed their understandings of what it means to teach for understanding. The paper begins with a discussion of what teaching for understanding might look like in practice. Next, the activities the students took part in are discussed, along with their reactions to these activities as outlined in the course evaluation and follow-up questionnaire. Next, evidence of their learning, as presented by the students themselves in their final paper is discussed. Finally, directions for similar courses in future are explored.

The purpose of this paper is to report the results of a study which explored how a group of 11 preservice secondary mathematics teachers developed their understandings of what it means to teach for understanding. This investigation is set in the context of the secondary mathematics education methods course¹ and practicum teaching experience which all 11 participants were undertaking. The paper begins with a discussion of the theoretical framework underpinning the course and its goals. Then some background to the study is provided. Next, the design and methodology of the study which is the subject of this paper is presented, followed by the results of the study. Finally, a discussion of the implications of the findings of this study for mathematics teacher education in general are discussed.

Background and Theoretical Framework

At Cleveland State University, the mission of the College of Education is to produce a teacher candidate who is "a responsive, reflective professional: a partner in learning". As developer of this course, the author had additional goals in mind, based on her own beliefs derived through 13 years of high school teaching and 5 years of doctoral research: that mathematics teaching must become more connected to the real world and be devoted to developing students' understanding of concepts as opposed to focusing on rote memorization (Goodell, 1998; Goodell & Parker, 2001). Thus, there were two foci for this course. The first was on developing the preservice teachers' understanding of the art of "teaching for understanding". The second was on facilitating their reflective thinking, and learning from that process.

Teaching for Understanding

Teaching for understanding has always been important in mathematics education, with recent publications on the subject by Hiebert, et al. (1997), Carpenter and Lehrer (1999), Fennema, Sowder and Carpenter (1999), and Sierpinska (1994). Current notions of teaching for understanding are largely based on constructivist

¹ In this paper, a course is a single subject, unit, or class. A group of courses which lead to a degree or certification will be referred to as a program.

principles. These principles have been loosely defined by Davis (1996) as the building up of the student's own knowledge, based on new input assimilated with information already known. As noted by Janvier (1996, p. 462) in a constructivist learning environment "each step from past knowledge to new knowledge is made by an extension through reflection". Several common threads run through all of this work: the notion that understanding in mathematics requires action on the part of the learner in the form of making connections to other things she or he already knows; that teachers have a critical role in promoting understanding through the ways in which they organise classroom instruction and assessment; and that reflection is a vital part of this process.

Carpenter and Lehrer (1999) propose five forms of mental activity from which mathematical understanding develops: constructing relationships, extending and applying mathematical knowledge, reflecting about experiences, articulating what one knows, and making mathematical knowledge one's own. One week early in the semester, the methods class meeting focused on the Carpenter and Lehrer (1999) article and the types of mathematics learning activities that fostered this kind of mental activity. Thereafter, all of the activities the preservice teachers participated in throughout the mathematics methods course were designed in order to reinforce these concepts. The activities included hands-on teaching and learning exercises led by the instructor; students teaching mini-lessons in the methods course meetings; discussing and writing reflections of assigned readings; researching and collecting relevant teaching resources; planning and teaching a two-week curriculum unit in their practicum school; being observed by the cooperating teacher and the instructor; discussing those observations after the lesson; and interacting with the cooperating teacher before teaching lessons.

Facilitating Reflection in Novice Teachers

The second major foci for this mathematics methods course was to facilitate preservice teachers' reflective thinking towards becoming a reflective practitioner. Loughran (1996) defined reflection as "the purposeful, deliberate act of inquiry into one's thought and actions through which a perceived problem is examined in order that a thoughtful, reasoned response might be tested out" (p. 21). As others have noted, most preservice teachers teach the way they were taught and through which they succeeded in learning, with a focus on rote memorization of procedures and facts (Brown & Borko, 1992; Goos, 1999; Janvier, 1996). Reformist ideals of constructivism and teaching for understanding are often not part of preservice teachers' prior experiences as learners of mathematics. Consistent with Cobb, Wood and Yackel's (1990) view that the only way to change the teaching practices of teachers is to help them see their current practices as problematic, a major component of this course was for each participant to write about a "critical incident" that happened to them or their cooperating teacher, and also to discuss these incidents in class. The written description of the critical incident had to describe why it happened, how the writer might have handled the situation differently, and what the implications for her or his practice might be in future.

Zeichner and Liston (1996) identified five traditions of reflective practice in teacher education during the 20th century: academic, social efficiency, developmentalist, social reconstructionist, and "generic" (p. 51). The critical incident

reflections followed the academic tradition in which “reflection on subject matter and the representation and translation of that subject matter to promote student understanding” (p. 51) is foremost.

Pultorak (1993) highlighted four reasons why novice teachers have difficulty in engaging in reflective thinking. These were educators’ lack of time and structured opportunities for reflection, their inability to look objectively at school-based experiences and benefit from them, school supervisors’ inadequate insight and enthusiasm, and university faculty members’ demanding workloads which may interfere with their attempts to ensure that teacher preparation programs foster opportunities for reflection. In order to encourage students to engage in the process of reflection, journals were assessed by allocating a significant proportion of the total points for the course to this component. Points were awarded for having attempted an entry each week, and not for the content of the entry. By making it a structured part of the course, all students completed their journals, although not always immediately after each meeting. Additionally, at the beginning of each weekly methods course meeting, the class shared their critical incidents with one another. This helped the preservice teachers overcome any lack of objectivity cited by Pultorak (1993) by having others comment critically on their reflections. As noted by Zeichner (1996) “There is very little sense, in a lot of the discourse on reflective teaching, of reflection as a social practice, where groups of teachers can support and sustain each other’s growth. (p.205)”. The use of critical incidents in both reflective journal writing and as a focus for class discussion was a deliberate attempt to address Zeichner’s concerns. Although class discussions regularly took much more time than originally intended by the instructor, these were typically of too rich a nature to warrant limitation.

In addition to the critical incident reflective journal, the preservice teachers were required to reflect on each methods class meeting, with a specific writing prompt: “What I learned this week; how I learned it; and what are the implications for my practice”. At the end of the semester, each student wrote a final paper in which she or he told their own individual story about what they had learned, how they learned it, and what the implications might be for them as teachers of mathematics. Both critical incident and weekly class journal entries, plus their experiences in their practicum placement were used as “data” for their papers. They also had to write their personal philosophy of teaching as their homework assignment after the first week of class, which they could then reflect on in their final paper. The following six questions were given to the students in the syllabus at the beginning of the semester, and were suggested as foci for the final paper.

1. What was your “philosophy of mathematics teaching” at the start of the semester?
2. How did it grow or evolve? What changes did you make in your teaching as a result of your systematic critical reflections on your teaching and as a result of your systematic reflective dialogue with your fellow teachers?
3. What were some of your main concerns, struggles, obstacles, questions, excitements, and “aha’s”? How did you resolve those obstacles or conflicts in your teaching? What are some of those conflicts that you are still searching to solve?

4. How did your participation in this dialogic community (your small group discussions) facilitate your professional development and growth?
5. What were some of the other influences on your professional growth this semester?
6. What is your personal philosophy of mathematics teaching and learning now?

The Students and the Teaching Certification Program

Only three of the preservice teachers who were part of this study were undergraduates pursuing a degree in mathematics: the remainder had completed a first degree in a variety of fields including law, accounting, engineering and art. All were undertaking courses in a secondary teaching certification program. The mathematics content requirements for secondary teaching certification were the same for both undergraduates and graduates; so those graduates whose undergraduate degrees were not in mathematics had to take a considerable number of mathematics content courses, thus ensuring that their mathematics content background was substantial. As part of their teaching certification program, the preservice teachers were all concurrently enrolled in a field experience of 180 contact hours, usually referred to as “practicum”, and a secondary mathematics methods course of 60 contact hours. They were required to attend their practicum school every week day morning for approximately two-and-a-half hours for the entire 15 week semester. Most of the time during their practicum was spent observing their cooperating teacher and assisting students. By the tenth week of semester, they were expected to teach a two week sequence of lessons to one of the two or three classes they had been observing. Most continued to teach that class until the end of the practicum. The methods course met once each week for four hours in the evening. Most preservice teachers also had other courses to take in the afternoons, as well as part time jobs and family responsibilities. In the semester following this course, the majority of preservice teachers undertook another field experience for a full 15 weeks known as “student teaching”, involving much more teaching, and usually in a different school to their practicum placement.

Research Questions and Methods of this Study

In this study, the research question was “how effective were all of the activities associated with the methods course and the practicum placement in developing the student teachers’ understanding of and expertise in teaching for understanding”. Permission was obtained from participants to use quotes from their journals and final papers, and each agreed to complete a questionnaire designed to gather information about the value of activities in helping them to come to their own understandings of what is meant by teaching for understanding. Thus, data sources consisted of the author’s own observations and journal entries, the students’ final papers, course evaluations, and responses to a follow-up questionnaire. This questionnaire was sent by mail in the semester following the methods course and practicum experience when the preservice teachers were undertaking their student teaching experience.

Quantitative data were summarised using measures of central tendency. As both the population and the sample were so small, further statistics were not calculated.

Qualitative data were entered into the software package NUD•IST (Qualitative Solutions and Research Pty Ltd, 1997) and emergent themes were identified and coded. The total respondents for each theme were calculated to give an indication of the relative importance of each theme as a vehicle for developing the preservice teachers' understanding of, and expertise in, teaching for understanding.

Findings

Themes in the Final Paper

At the end of the semester, students were required to write a paper summarising their experiences in the methods course and practicum. The background to this is discussed earlier in this paper. Five issues were mentioned as major areas of professional growth by a majority of students (i.e., 6 or more students out of the group of 11 wrote about it), namely, the value of the weekly discussions of critical incidents, incorporating hands-on activities to promote understanding, the difficulty of ensuring understanding, the benefits of using multiple methods of presentation, and the importance of connecting with students' interests and backgrounds.

Weekly discussions. The first issue was the value of the weekly discussions with their peers. Alan's² comment illustrates this theme:

My fellow student colleagues were also a real source of growth for me. I found the small group discussions in the classroom to be incredibly helpful. I look forward to them every week.

These discussions were often centred around issues the students were facing in their practicum teaching, and were usually generated at the start of each class through the sharing of critical incidents. The range of issues discussed was immense, and as previously noted, the discussion often took a significant portion of the class meeting time. The issues discussed included classroom management and school discipline policies; legal responsibilities of teachers; mathematics pedagogy, assessment and curriculum issues; issues of gender, race and culture and how these affect students and teachers in the mathematics classroom; student motivation; the impact of absenteeism; dealing with students' personal problems; teaching students with disabilities; and issues associated with large urban districts such as poverty and crime. The most noticeable aspect of these discussions from the author's view point was that it gave the students the opportunity to connect the theories they had learned in the many different courses they had completed—such as educational psychology, social issues, general teaching methods, and the current study of mathematics teaching methods—to their real-life experiences in the classroom. Articles and books they had read were often mentioned during the discussion to try to explain or understand the situation. The author (instructor) confined herself to the role of observer, only commenting at the end or when asked by the students. In this way, the group came to see itself as a learning community, a supportive group on whom they could depend to understand the issues they were facing.

² All names used are fictitious

Incorporating hands-on activities to promote understanding. The second theme identified in the students' final papers was the importance of incorporating hands-on activities as noted by Elizabeth:

I have come to believe that math can be taught more effectively by incorporating hands-on and real life applications into the classroom than through traditional lecturing.

As mentioned previously, weekly methods class activities were designed to illustrate these concepts to the preservice teachers. For example, when examining ways of teaching measurement concepts, the class undertook an activity which led them to "discover" pi. In this activity, students measured the diameter and circumference of a variety of different sized circular objects using string, tape measures and rulers. They then calculated the ratio of the circumference to the diameter and generalised their discovery to write a rule for this process. Many of the preservice teachers had never seen or participated in this activity, or anything like it, and were amazed at their "discovery". Some subsequently tried the same or similar activity with their own practicum classes with varying degrees of success, which naturally led to very fruitful critical incident discussions. In this way, through demonstration and experimentation with their own classes, most of the preservice teachers gradually came to value students constructing their own knowledge over more traditional knowledge transmission methods of teaching. However, not all preservice teachers came to this same understanding. Even those who did expressed frustration at trying to implement constructivist teaching practices all the time, as is illustrated by some of the written comments in the course evaluations and follow-up questionnaire presented later in this paper.

Ensuring understanding. The third major theme mentioned in the final papers was the difficulty of teaching for understanding. As previously mentioned, the preservice teachers had all experienced very traditional teaching as mathematics students. Teaching for understanding was a new concept for them, and they were experiencing many barriers in their quest to ensure understanding in their own students. For example, as Helen noted in the quote below, there is always time pressure to "cover" the curriculum, whether or not all students have understood the concepts:

Being exposed to something and understanding something are two different things. ...the pressure to "go on" even though many students are clearly lost is a force to be reckoned with. I struggle with what you do when 20% of the class does not understand the given concept and the other 80% is beginning to get bored.

Additionally, much more planning time is needed to create an active learning environment that fosters understanding, as noted by Elizabeth:

I have also come to learn that creating a classroom that includes these various activities and teaching concepts requires significant planning.

Time pressures on all teachers are considerable, and for beginning teachers, even greater. Finding ways to support beginning teachers so that they have adequate time to plan properly is an important issue addressed in the Discussion section of this paper.

Multiple methods of presentation. The fourth theme identified in the final papers was the importance of using multiple methods of teaching in order to enhance connections with students, as noted by Rodger in the following quote:

The benefits of hands-on learning are much clearer to me. ... Plus providing a variety of learning opportunities has the greatest chance to connect with the greatest number of students.

Michael also commented on the importance of being creative in planning and presenting lessons to enhance student participation and learning:

Creativity is appreciated and enjoyed by the students. My more creative lessons resulted in more participation and higher scores than traditional lecture lessons did. Also, when I used creativity, class disruptions decreased because the students were more focused on the lessons.

This kind of realization and meaningful learning by preservice teachers can only occur through personal experience. Michael's first few lessons, being not so "creative", were not very successful. He was ready to hear the messages his cooperating teacher (who was a highly experienced and successful teacher whose teaching fostered the Carpenter and Lehrer (1999) ideals discussed earlier), his classmates and the author were giving him about how to improve his lessons. He demonstrated considerable growth throughout the semester in his understanding of teaching for understanding.

Connecting with students. The fifth issue discussed by many preservice teachers in their final papers concerned the importance of connecting with students interests and backgrounds, as noted by Olivia:

When students are presented with a context in which to explore and make connections the exploration has more meaning. ... Concepts need to be presented in such a way as to have relevance for the students, and students' varied backgrounds ethnically, educationally and socially need to be understood to provide them with the best opportunity to learn.

This issue came up repeatedly in the methods class sessions during discussions of critical incidents that involved students' lack of motivation, and during the close examination of the National Council of Teachers of Mathematics (NCTM)(2000) standards documents³. Many of the preservice teachers acknowledged that their own experiences in learning mathematics lacked this kind of connection, and were often abstract and unrelated to anything students might find interesting. However, this is often less of an issue for future secondary teachers as they are typically successful in these traditional mathematics courses, as opposed to the majority of preservice elementary teachers who often cite the lack of connection with the real world as one reason for their dislike of mathematics. Nevertheless, all of the participants in this course attempted to find real-world contexts for their own students throughout their practicum experience. This was reflected in their selection of learning resources for their lesson presentations during the methods class and for the two-week unit they wrote for their practicum class (author's journal).

³ The draft of NCTM standards document was used as the 2000 edition was not available when this course was offered.

Other issues. Students also mentioned a range of other areas in which they had experienced growth throughout the semester. These included assessment issues such as fairness and use of multiple methods, the importance of adequate lesson planning, learning to be flexible and respond to unexpected things in lessons, and classroom management issues. As noted previously, the connections between theory and practice were being made concrete.

Follow-up Questionnaire

In the spring (second) semester, all of the students were again out in the field completing their final student teaching requirements. This consisted of a 15 week full-time assignment, with teaching loads building up to almost full-time (4 or 5 classes) by the eighth week. Follow-up questionnaires were mailed in March (at about the half-way point) so as to allow enough time for the preservice teachers to implement some of the approaches discussed in the methods course. Of the 11 sent, only 5 were returned, no doubt a reflection of the timing.

Activities rating questions. All activities undertaken in the methods courses were listed, and respondents were asked to rate each twice. The first rating concerned how helpful that activity was in developing the student's understanding of teaching for understanding. Five responses were possible: very helpful, somewhat helpful, neutral, somewhat unhelpful, and very unhelpful. These responses were given a numerical value of 5 through 1, respectively, and mean scores were calculated. The higher the mean score, the more helpful students perceived this activity to be.

The second rating concerned how often that activity should occur in future mathematics methods courses compared to their own experience. There were three responses possible for this: more often, the same, and less often. These were given a numerical value of 3 through 1, respectively. The higher the mean score, the more students suggested this activity should be used in future.

Table 1 shows the mean responses for the five students on each of the questions in the follow-up questionnaire. From this table, it can be seen that the activity these five students thought was least helpful was the reflection on the assigned weekly reading. They also indicated that this should occur less often in future. This was not too surprising given that this course was very practically oriented, and that this component was not weighted highly in the assessment schedule.

Table 1
Mean scores on follow-up questionnaires

| Activities in EDS 315/515 | Mean helpfulness | Mean use in future |
|--|------------------|--------------------|
| 1. Keeping a weekly reflective journal. | 3.8 | 2.0 |
| 2. Writing reflections about readings. | 3.4 | 1.2 |
| 3. Keeping a weekly critical incident journal. | 4.6 | 2.4 |
| 4. Discussing critical incidents in class. | 5.0 | 2.6 |
| 5. Writing self-evaluation after each lesson. | 3.6 | 1.8 |
| 6. Discussing lessons with cooperating teacher before teaching. | 4.2 | 2.2 |
| 7. Discussing lessons with cooperating teacher after teaching. | 4.2 | 2.2 |
| 8. Discussing lessons with professor before teaching. | 4.4 | 2.4 |
| 9. Discussing lessons with professor after teaching. | 4.8 | 2.4 |
| 10. Teaching mini lessons in Tuesday night class. | 3.6 | 2.0 |
| 11. Observing peers teaching mini lessons in Tuesday night class. | 3.8 | 2.0 |
| 12. Engaging in hands-on activities in Tuesday night class. | 4.2 | 2.0 |
| 13. Writing final paper about influence of practicum on professional growth. | 4.2 | 2.0 |

Clearly the most useful activity for these five students was the critical incident discussion which occurred each week in class. All five said this was very helpful, and three out of five said this should occur more often in future.

Open ended question responses. These questions enabled students to elaborate their thoughts about the value of the activities undertaken in this methods course. For the most part, they were happy with the activities but had a few suggestions that will be implemented in the future. Two responses to the question "What other activities would have helped you then, or are helping you now, to develop your ideas about teaching for understanding?" are given below:

I think it would be helpful for us to present a lesson we are planning to teach to our classmates - maybe even in groups and then coming back to the group the next week with reflections on how it went, what worked, what didn't, why, what we'd do differently. My methods class was one of my best courses to help me prepare to be a teacher. (Elizabeth, March 2000)

Ways to better assess student comprehension, and understanding and interest in materials. I rely heavily on informal assessment during my lessons. (Rodger, March 2000)

One student in particular (Alan) was quite resistant to the ideas of teaching for understanding during the methods course. He would often make comments like "why isn't there a book outlining the best way to teach every concept" (author's

journal). He also commented in his final paper that “I naively thought that if the teacher understood the material, and taught from the curriculum, that learning would occur. I had been taught under this system and I had done very well in math”. His responses to the open ended questions revealed significant growth beyond these earlier comments, but also indicated that he was “still struggling”. His response to the question “What other activities would have helped you then, or are helping you now, to develop your ideas about teaching for understanding?” is given below.

I’m still struggling with the idea of “teaching for understanding.” My kids don’t respond to it! They like the “recipe” approach to math: give them steps to follow and they do it. But they need the understanding. I think we were successful doing “teaching for understanding” activities in EDS 515 because we already have math backgrounds and skills. I’m finding it a hard sell in low-achieving classrooms. (Alan, March 2000)

The next question “Which of these activities have continued into your student teaching experience?” again reveals his struggle and desire to remain open-minded about the value of teaching for understanding:

Ironically, I’m most interested in using hands-on activities in my classroom and having students “discover” math. We occasionally do calculator exploration labs. I’m not sure how much understanding takes place, but students stay on task. Bottom line I believe students should understand the math they are studying. The question in my mind is can they? (Alan, March 2000)

Naturally, this was not “ironic” at all to the author: Alan had just validated the whole course and its approach. Every activity was designed to emphasise that teaching for understanding was the main goal of mathematics teaching, and his comment clearly indicated that he received that message. The fact that he is struggling to implement these ideas is also no surprise, although of major significance for the mathematics teaching profession, to be discussed in the next section.

Discussion and Implications

Facilitators of Learning to Teach for Understanding

The discussion of critical incidents in methods class sessions was the most important element identified by the preservice teachers for helping to develop their understanding of teaching for understanding. This was mentioned by a majority of students in the final paper, as well as by all five respondents in the follow-up questionnaire. Writing the critical incident journal was also an important element in facilitating the reflective process, and was mentioned by many students. As noted earlier, a constructivist learning environment must incorporate reflection (Janvier, 1996). The findings of this study clearly demonstrate the value of critical incidents journals and discussions in this process. In this instance, the process of reflection enabled the preservice teachers to construct their own understanding of the value of teaching mathematics for understanding. It should be noted that one strength of this program that made the process of reflection possible was the concurrent enrollment of the preservice teachers in the methods course and field experience components.

All too often, methods courses present theories of learning, and techniques to implement these theories, in isolation from the real world. We preach constructivist philosophies while not practicing them ourselves. Preservice teachers often have no opportunity to put these ideas into practice until some time later.

In this program however, preservice teachers had an opportunity to try out the suggestions and techniques demonstrated in class each week, and then were able to discuss with their peers their successes and failures, although, as indicated earlier in a comment made by Elizabeth on how to improve the course in future, she would have liked to have even more opportunities to do this. Nevertheless, these preservice teachers had the opportunity to construct their own knowledge about what is effective teaching, with the scaffolding supportive peer group discussions, and the motivation of being assessed on their reflections. After having taught a number of courses structured in this way, this author cannot contemplate teaching a methods course without a concurrent field experience component. All teacher education programs would benefit from such linkages.

The other activity that the majority of students found “very helpful” or “helpful” was discussing a lesson with the cooperating teacher or instructor after teaching it. This kind of activity has long been a component of teacher education, although with varying degrees of success (Borko & Mayfield, 1995). There are two main reasons why these preservice teachers found this aspect of their practicum to be very worthwhile. Firstly, the cooperating teachers were mostly all known by the author or other university faculty, and were chosen because of their superior teaching skills, knowledge of reform, or involvement in the university’s masters degree program. Secondly, the author, as course developer, was also the university supervisor, hence there was no confusion as to the goals of their experience: everyone was on the same page. Whenever possible, both the cooperating teacher and the author would conference with the preservice teacher about the lesson directly after its completion. Comprehensive written feedback was also provided to enable further reflection after the conferencing was complete.

It should also be noted that those activities identified by the preservice teachers as influential in helping them develop their understanding of teaching for understanding involve reflective thinking, writing and discussing critical incidents, and post lesson conferencing, confirming the author’s original idea and the findings of Loughran (1996) that reflection would play an important role in preservice teachers learning to teach. It is this self-constructed knowledge that will hopefully enable these preservice teacher to continue to strive to teach for understanding as they enter their teaching careers. As noted by Loughran (1996), teachers who have developed the art of reflective practice are much more likely to learn from their experience—they will have ten years of experience rather than having one year of experience ten times over. However, they will undoubtedly face some barriers along the way. These will be discussed next, along with suggestions as to how to minimize them.

Barriers to Implementing Teaching for Understanding.

One important question to consider when trying to develop the “art” of teaching for understanding in preservice teachers is “how do we know when students understand”. This is a central question for mathematics teacher educators: “What

kind of evidence is needed to demonstrate understanding?" The issue of assessment came up many times in the critical incident discussions. The importance of using multiple assessment instruments, and requiring different forms of representation including oral, written, diagrammatic, and traditional pencil and paper methods were emphasized, although as noted earlier by Rodger, he would have liked more guidance on this.

Related to this is the issue raised by Helen of what she should do when it is clear that many of the students didn't understand, but the time pressure to go on is intense. In the context of the United States educational system, with the current climate of high-stakes multiple-choice tests being used to reward high-performing students, teachers and districts, and punishing low-performing ones (Popham, 1999), the pressure to "cover the content before the test" manifests itself in "drill and kill" activities for considerable periods of time: in some cases, up to 6 weeks before the test (Goodell, Parker & Kahle, 2000). The consequence is that teachers often view developing understanding in opposition to preparing students for high-stakes tests, and this tension is certainly also felt by preservice teachers in the field.

Alan's responses to the open-ended questions highlight the need for ongoing support in the first years of teaching. Alan is not yet convinced that teaching for understanding is the best way to teach his lower-achieving students. Without an ongoing professional forum in which to receive support for his efforts to implement teaching for understanding, it may well be that Alan will abandon his attempts, that is, revert to the chalk and talk methods by which he was taught. This barrier will be exacerbated in schools with outdated curriculum materials that do not support the reform agenda, forcing beginning teachers, who are under time pressure just learning to cope with a new job and many requirements other than teaching, to spend considerable time preparing lessons, trying to make their lessons more real-world or creative.

Beginning teachers should not be assigned the most difficult classes to teach in their first year, and should also be given a reduced load to ease time pressures and help them have a successful first year. Additionally, access to high quality up-to-date resources can greatly assist beginning teachers in their early years. Unfortunately, in many school systems in the United States, access to current reform curriculum materials cannot be assured. It is important therefore for mathematics teacher educators to reassure their graduates that it is not necessary (or desirable) to try to change the world (their school's mathematics department) overnight. Rather, a gradual integration of the new ideas they are bringing with them with the old ideas established in their school is more likely to be successful.

Conclusion

In future, in mathematics education methods courses of a similar nature, journals ought to be collected on a more regular basis, to encourage people to keep up to date with their entries. More time ought to be spent introducing the elements of reflective thinking and preservice teachers should be required to provide more evidence of changes they have made or intend to make as a result of their reflections. Additionally, more time should be spent on assessment issues to ensure that the preservice teachers have a good grasp of not only how to develop understanding in their students, but also how to assess their progress in doing so.

Learning to teach is a complex process, and novice teachers need a lot of support in the early stages. This research has confirmed that providing structured opportunities for reflection in the form of journals, time for discussion of critical incidents in class, and discussions of teaching episodes with cooperating teachers and professors are important elements in learning to teach for understanding. Equally important is a linkage between the methods course and practicum experience, accompanied by quality supervision by cooperating teachers and university faculty, and subsequent ongoing support in the early years of teaching.

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Studies Teacher Education, Mathematics Education, and Reflective Teaching. Mathematics teacher educator and director of the Center for Faculty Excellence. • Learning to Teach Mathematics for Understanding: The Role of Reflection more. by Joanne E Goodell. This paper explores how a group of 11 preservice secondary mathematics teachers developed their understandings of what it means to teach for understanding. The paper begins with a discussion of what teaching for understanding might look more. This paper explores how a group of 11 preservice secondary mathematics teachers developed their understandings of what it means to teach for understanding.