

EXPERIENCING GEOMETRY

EUCLIDEAN AND NON-EUCLIDEAN
WITH HISTORY

THIRD EDITION

David W. Henderson

Daina Taimiņa

Cornell University, Ithaca, New York



Upper Saddle River, New Jersey 07458

CONTENTS

Preface	xv
Changes in this Edition	xvii
Useful Supplements	xviii
Our Background in Geometry	xix
Acknowledgments for the First Edition	xxi
Acknowledgments for the Second Edition	xxii
Acknowledgments for This Edition	xxiii
How to Use This Book	xxv
How We Use this Book in a Course	xxvi
But, Do It Your Own Way	xxvii
Chapter Sequences	xxviii
Chapter 0 Historical Strands of Geometry	1
Art/Patterns Strand	1
Navigation/Stargazing Strand	3
Building Structures Strand	4
Motion/Machines Strand	6

Chapter 1 What Is Straight?	9
History: How Can We Draw a Straight Line?	9
PROBLEM 1.1 When Do You Call a Line Straight?	13
The Symmetries of a Line	17
Local (and Infinitesimal) Straightness	21
Chapter 2 Straightness on Spheres	25
Early History of Spherical Geometry	25
PROBLEM 2.1 What Is Straight on a Sphere?	28
Symmetries of Great Circles	32
*Every Geodesic Is a Great Circle	35
*Intrinsic Curvature	35
Chapter 3 What Is an Angle?	37
PROBLEM 3.1 What Is an Angle?	37
PROBLEM 3.2 Vertical Angle Theorem (VAT)	39
Hints for Three Different Proofs	41
Chapter 4 Straightness on Cylinders and Cones	43
PROBLEM 4.1 Straightness on Cylinders and Cones	44
Cones with Varying Cone Angles	46
Geodesics on Cylinders	49
Geodesics on Cones	50
*PROBLEM 4.2 Global Properties of Geodesics	51
* n -Sheeted Coverings of a Cylinder	51
* n -Sheeted (Branched) Coverings of a Cone	53
Locally Isometric	55
Is “Shortest” Always “Straight”?	56
*Relations to Differential Geometry	57
Chapter 5 Straightness on Hyperbolic Planes	59
A Short History of Hyperbolic Geometry	59
Description of Annular Hyperbolic Planes	62
Hyperbolic Planes of Different Radii (Curvature)	64
PROBLEM 5.1 What Is Straight in a Hyperbolic Plane?	66

*PROBLEM 5.2	Coordinate System on Annular Hyperbolic Plane	63
*PROBLEM 5.3	The Pseudosphere Is Hyperbolic Intrinsic/Extrinsic, Local/Global	68 71
PROBLEM 5.4	Rotations and Reflections on Surfaces	71
Chapter 6 Triangles and Congruencies		73
*Geodesics Are Locally Unique		73
PROBLEM 6.1	Properties of Geodesics	74
PROBLEM 6.2	Isosceles Triangle Theorem (ITT) Circles	75 76
	Triangle Inequality	78
PROBLEM 6.3	Bisector Constructions	79
PROBLEM 6.4	Side-Angle-Side (SAS)	80
PROBLEM 6.5	Angle-Side-Angle (ASA)	85
Chapter 7 Area and Holonomy		89
PROBLEM 7.1	The Area of a Triangle on a Sphere	90
PROBLEM 7.2	Area of Hyperbolic Triangles	91
PROBLEM 7.3	Sum of the Angles of a Triangle Introducing Parallel Transport	95 96
	Introducing Holonomy	98
PROBLEM 7.4	The Holonomy of a Small Triangle	100
	The Gauss-Bonnet Formula for Triangles	102
*PROBLEM 7.5	Gauss-Bonnet Formula for Polygons	103
*Gauss-Bonnet Formula for Polygons on Surfaces		106
Chapter 8 Parallel Transport		109
PROBLEM 8.1	Euclid's Exterior Angle Theorem (EEAT)	109
PROBLEM 8.2	Symmetries of Parallel Transported Lines	111
PROBLEM 8.3	Transversals through a Midpoint	114
PROBLEM 8.4	What Is "Parallel"?	115
Chapter 9 SSS, ASS, SAA, and AAA		117
PROBLEM 9.1	Side-Side-Side (SSS)	117
PROBLEM 9.2	Angle-Side-Side (ASS)	119
PROBLEM 9.3	Side-Angle-Angle (SAA)	121
PROBLEM 9.4	Angle-Angle-Angle (AAA)	123

Chapter 10	Parallel Postulates	125
	Parallel Lines on the Plane Are Special	125
	PROBLEM 10.1 Parallel Transport on the Plane	126
	PROBLEM 10.2 Parallel Postulates Not Involving (Non-)Intersecting Lines	128
	Equidistant Curves on Spheres and Hyperbolic Planes	130
	PROBLEM 10.3 Parallel Postulates Involving (Non-)Intersecting Lines	131
	PROBLEM 10.4 EFP and HSP on Sphere and Hyperbolic Plane	134
	Comparisons of Plane, Spheres, and Hyperbolic Planes	136
	Parallel Postulates within the Building Structures Strand	138
	Non-Euclidean Geometries within the Historical Strands	140
Chapter 11	Isometries and Patterns	143
	PROBLEM 11.1 Isometries	144
	PROBLEM 11.2 Three Points Determine an Isometry	148
	PROBLEM 11.3 Classification of Isometries	149
	Klein's Erlangen Program	153
	Symmetries and Patterns	154
	PROBLEM 11.4 Examples of Patterns	158
	*PROBLEM 11.5 Classification of Discrete Strip Patterns	159
	*PROBLEM 11.6 Classification of Finite Plane Patterns	159
	*PROBLEM 11.7 Regular Tilings with Polygons	160
	*Other Periodic (and Non-Periodic) Patterns	161
	*Geometric Meaning of Abstract Group Terminology	163
Chapter 12	Dissection Theory	165
	What Is Dissection Theory?	165
	A Dissection Puzzle from 250 B.C. Solved in 2003	167
	History of Dissections in the Theory of Area	168
	PROBLEM 12.1 Dissect Plane Triangle and Parallelogram	169
	Dissection Theory on Spheres and Hyperbolic Planes	170
	PROBLEM 12.2 Khayyam Quadrilaterals	171
	PROBLEM 12.3 Dissect Spherical and Hyperbolic Triangles and Khayyam Parallelograms	172
	*PROBLEM 12.4 Spherical Polygons Dissect to Lunes	173

Chapter 13	Square Roots, Pythagoras, and Similar Triangles	177
	Square Roots	178
	PROBLEM 13.1 A Rectangle Dissects into a Square	179
	Baudhayana's Sulbasutram	184
	PROBLEM 13.2 Equivalence of Squares	189
	Any Polygon Can Be Dissected into a Square	190
	History of Dissections	191
	PROBLEM 13.3 More Dissection-Related Problems	193
	*Three-Dimensional Dissections and Hilbert's Third Problem	194
	PROBLEM 13.4 Similar Triangles	195
Chapter 14	Projections of a Sphere onto a Plane	197
	PROBLEM 14.1 Charts Must Distort	198
	PROBLEM 14.2 Gnomonic Projection	198
	PROBLEM 14.3 Cylindrical Projection	199
	PROBLEM 14.4 Stereographic Projection	200
	History of Stereographic Projection and Astrolabe	202
Chapter 15	Circles	205
	PROBLEM 15.1 Angles and Power of Points for Circles in the Plane	206
	*PROBLEM 15.2 Power of Points for Circles on Spheres	208
	PROBLEM 15.3 Applications of Power of a Point	212
	PROBLEM 15.4 Trisecting Angles and Other Constructions	213
Chapter 16	Inversions in Circles	217
	Early History of Inversions	217
	PROBLEM 16.1 Inversions in Circles	218
	PROBLEM 16.2 Inversions Preserve Angles and Preserve Circles (and Lines)	221
	PROBLEM 16.3 Using Inversions to Draw Straight Lines	224
	*PROBLEM 16.4 Apollonius' Problem	226
	Expansions of the Notion of Inversions	230

Chapter 17	Projections (Models) of Hyperbolic Planes	233
	Distortion of Coordinate Systems	234
	*PROBLEM 17.1 A Conformal Coordinate System	236
	*PROBLEM 17.2 Upper Half-Plane Is Model of Annular Hyperbolic Plane	237
	PROBLEM 17.3 Properties of Hyperbolic Geodesics	239
	PROBLEM 17.4 Hyperbolic Ideal Triangles	241
	PROBLEM 17.5 Poincaré Disk Model	242
	PROBLEM 17.6 Projective Disk Model	244
Chapter 18	Geometric 2-Manifolds	245
	PROBLEM 18.1 Flat Torus and Flat Klein Bottle	246
	*PROBLEM 18.2 Universal Covering of Flat 2-Manifolds	251
	PROBLEM 18.3 Spherical 2-Manifolds	253
	*Coverings of a Sphere	256
	PROBLEM 18.4 Hyperbolic Manifolds	258
	PROBLEM 18.5 Area and Euler Number	263
	*Triangles on Geometric Manifolds	264
	PROBLEM 18.6 Can the Bug Tell Which Manifold?	265
Chapter 19	Geometric Solutions of Quadratic and Cubic Equations	267
	PROBLEM 19.1 Quadratic Equations	268
	PROBLEM 19.2 Conic Sections and Cube Roots	272
	PROBLEM 19.3 Solving Cubic Equations Geometrically	276
	PROBLEM 19.4 Algebraic Solution of Cubics	280
	What Does This All Point To?	282
Chapter 20	Trigonometry and Duality	285
	PROBLEM 20.1 Circumference of a Circle	285
	PROBLEM 20.2 Law of Cosines	287
	PROBLEM 20.3 Law of Sines	290
	Duality on a Sphere	292
	PROBLEM 20.4 The Dual of a Small Triangle on a Sphere	293
	PROBLEM 20.5 Trigonometry with Congruences	294
	Duality on the Projective Plane	294
	*PROBLEM 20.6 Properties on the Projective Plane	296

Cosmic Background Radiation	356
PROBLEM 24.5 Circle Patterns May Show the Shape of Space	360
Latest Evidence on the Shape of Space	361
Appendix A Euclid's Definitions, Postulates, and Common Notions	363
Definitions	363
Postulates	366
Common Notions	366
Appendix B Constructions of Hyperbolic Planes	367
The Hyperbolic Plane from Paper Annuli	367
How to Crochet the Hyperbolic Plane	368
$\{3, 7\}$ and $\{7, 3\}$ Polyhedral Constructions	371
Hyperbolic Soccer Ball Construction	371
" $\{3, 6\frac{1}{2}\}$ " Polyhedral Construction	372
Bibliography	375
Index	385

Geometry is defined as the experience of a person's field of vision becoming partially or completely encompassed by fast-moving, colorful, and indescribably complex geometric patterns, [1][2][3][4][5][6][7] form constants, [8][9] shapes, [4] fractals, [4] and colors. These geometric forms can also become structured and organized in a manner that appears to present genuine information to the person experiencing them far beyond the perception of meaningless, although complex, shapes, and colors. The geometric representations may Experiencing Geometry. Average rating: 0 out of 5 stars, based on 0 reviews Write a review. This button opens a dialog that displays additional images for this product with the option to zoom in or out. Tell us if something is incorrect. Experiencing Geometry. Average rating: 0 out of 5 stars, based on 0 reviews Write a review. \$107.80 \$107.80 \$107.80 \$107.80. Manufacturers, suppliers and others provide what you see here, and we have not verified it. See our disclaimer. 9780131437487 Experiencing Geometry. Specifications. Publisher.